Symmetric Random Butterfly Transform (SRBT) Based Preconditioner

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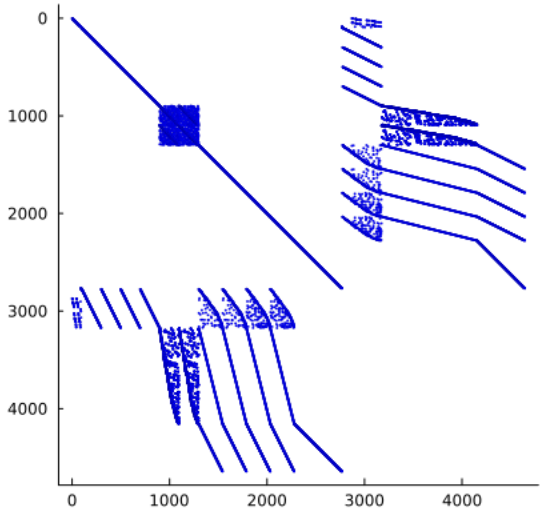
Motivation

- Interior Point Optimization requires repeated solutions of linear systems
- Matrices are
  - Sparse
  - Indefinite
  - Ill-conditioned
- Current parallel solvers perform poorly (Świrydowicz, et al. 2022)



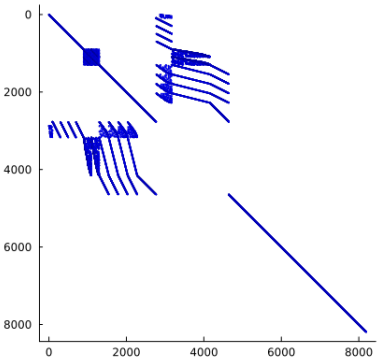
KKT matrix sparsity pattern from IPOPT run of ACTIVSg200 test system (Birchfield 2016)

Symmetric Random Butterfly Transform

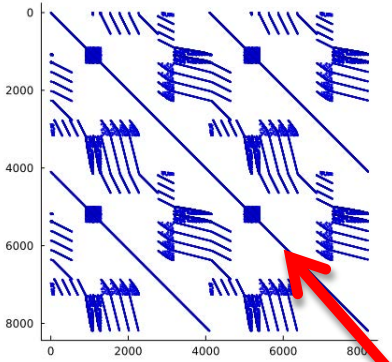
Originally proposed in (Parker and Lê 1995) and additional work by (Baboulin 2012)

KEY IDEA: Replace pivoting with multiplication by a random matrix

$$U^T \times$$



$$\times\ U =$$



Special Random Matrix

No Pivoting Required!

Symmetric Random Butterfly Transform

Definition: An *n*-by-*n* **Butterfly Matrix** is

$$B = \frac{1}{\sqrt{2}} \begin{pmatrix} R & S \\ R & -S \end{pmatrix}$$

where *R* and *S* are random, diagonal, nonsingular *n*/2-by-*n*/2 matrices.

Symmetric Random Butterfly Transform

An *n*-by-*n* **Recursive Butterfly Matrix** is of depth *d* is

$$U = U_d \times \cdots \times U_1,$$

where

$$U_k = \begin{pmatrix} B_1 & & \\ & \ddots & \\ & & B_{2^{k-1}} \end{pmatrix}$$

and each submatrix *B* is a butterfly matrix.

Note: This definition requires that $n = 2^m$ for some *m*.

Symmetric Random Butterfly Transform

To solve

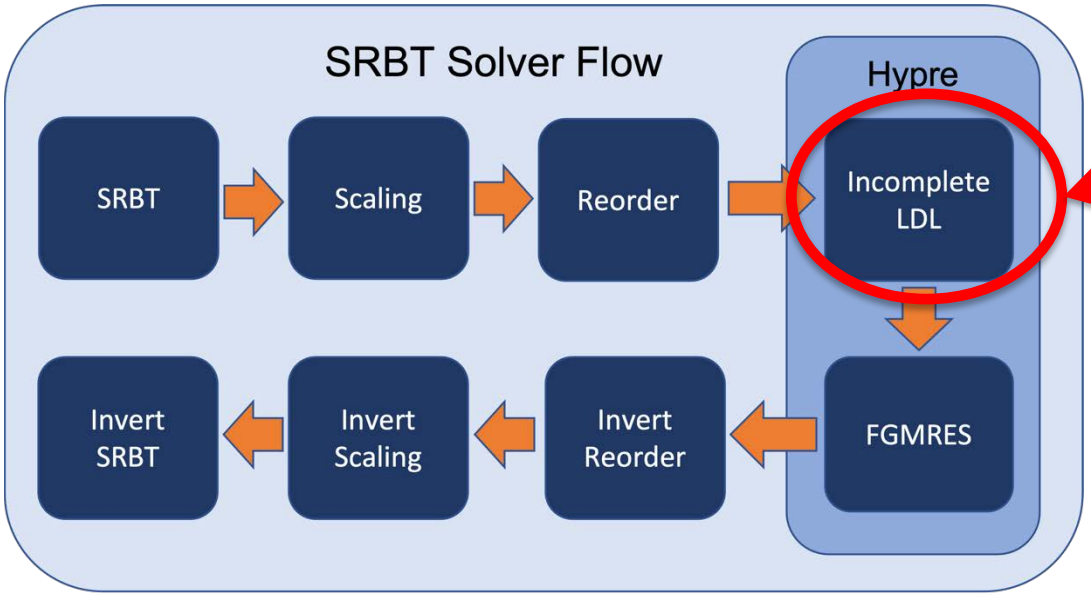
$$Ax = b$$

we use the Symmetric Random Butterfly Transform (SRBT):

$$\begin{aligned} A_r &= U^T A U \\ c &= U^T b \end{aligned} \quad \Rightarrow \quad A_r y = c \quad \Rightarrow \quad x = Uy$$

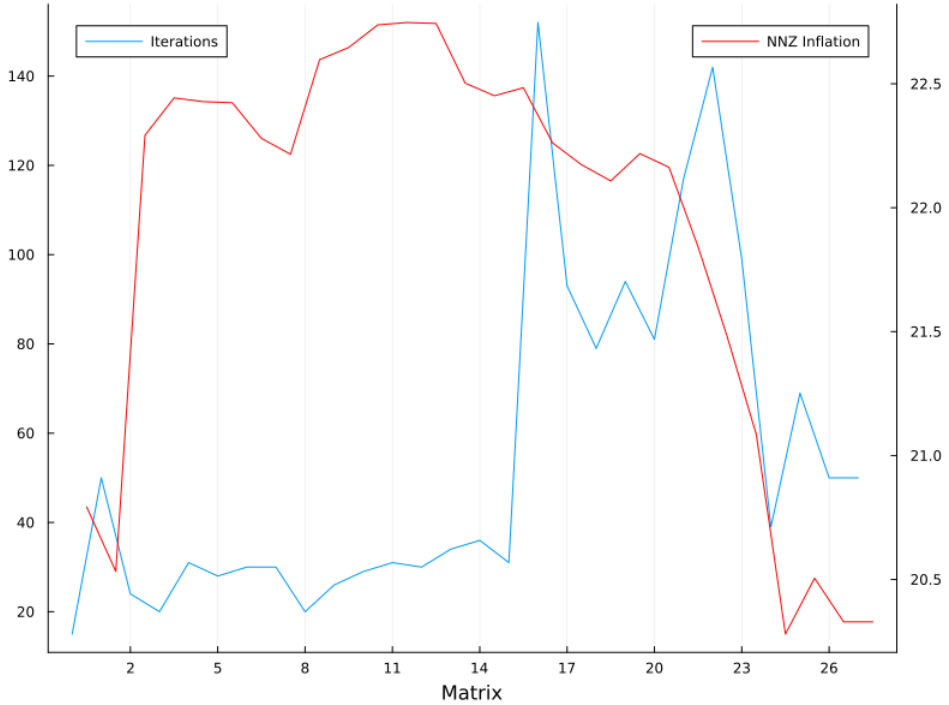
For more details see (Parker and Lê 1995) and (Baboulin 2012)

Full GPU Resident Solver



We use the Incomplete LDL from (Grief et al 2017) and FGMRES from (Świrydowicz et al 2021)

Results



Matrices available at: https://github.com/NREL/opf_matrices

$$NNZ\ Inflation = \frac{NNZ(ILDL)}{NNZ(A)}$$

Parameters Used

- SRBT
  - $d = 1$
  - $U_{ij} = e^{(\rho/10)}$, $\rho \sim U[-1/2, 1/2]$
- ILDL
  - *drop tol = 1e-12*
  - *fill factor = 20*
- *FGMRES*
  - *tolerance = 1e-8*
  - *krylov dim = 200*

Needed Work

1) Probability estimates of SRBT success based on depth
2) Better understanding of when this method works and when it fails
3) Quality GPU implementation of ILDL to benchmark
4) Remove requirement that $n = 2^m$

Collaborators

Thanks to

Stephen Thomas (AMD)

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References

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Thank you!
Questions?

www.nrel.gov

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